Корчагин Валерий Владимирович

Тюменский государственный университет

Институт Математики и Компьютерных Наук

Кафедра программной и системной инженерии

Студент бакалавриата

Группа 22МиР1710-1

korchagin8.valeriy@gmail.com

Сухарева Ольга Эдуардовна

Тюменский государственный университет

Институт математики и Компьютерных Наук

Кафедра иностранных языков и межкультурной

профессиональной коммуникации

Доцент, канд. фил. Наук

o.e.sukhareva@utmn.ru

ФРАКТАЛЬНАЯ ГРАФИКА

Korchagin Valeriy Vladimirovich

University of Tyumen

Institute of Mathematics and Computer Science

Software and Systems Engineering Department

Student of 22MaR1710-1 gr.

korchagin8.valeriy@gmail.com

Sukhareva Olga Eduardovna

University of Tyumen

Institute of Mathematics and Computer Science
Foreign Languages and Intercultural
Professional Communication Department
Associate Professor, Candidate of Philological Sciences
o.e.sukhareva@utmn.ru

FRACTAL GRAPHICS

Аннотация. Цель статьи - дать читателю некоторую информацию о фракталах и компьютерной графике. Большое внимание уделяется истории фракталов и алгоритмам создания фракталов. На мой взгляд, статья довольно интересна, потому что она показывает уникальную красоту компьютерной графики, которая генерируется фракталами.

Ключевые слова: Фрактал, Самоподобие, Компьютерная графика, Фрактальная графика, Кривая Коха, Бенуа Мандельброт.

Abstract. The purpose of the article is to give the reader some information on fractals and computer graphics. Much attention is given to the history of fractals and the algorithms of creating fractals. In my opinion the article is rather interesting because it shows the unique beauty of computer graphics which is generated by fractals.

Key words: Fractal, Self-Similarity, Computer Graphics, Fractal Graphics, Koch snowflake, Benoit Mandelbrot.

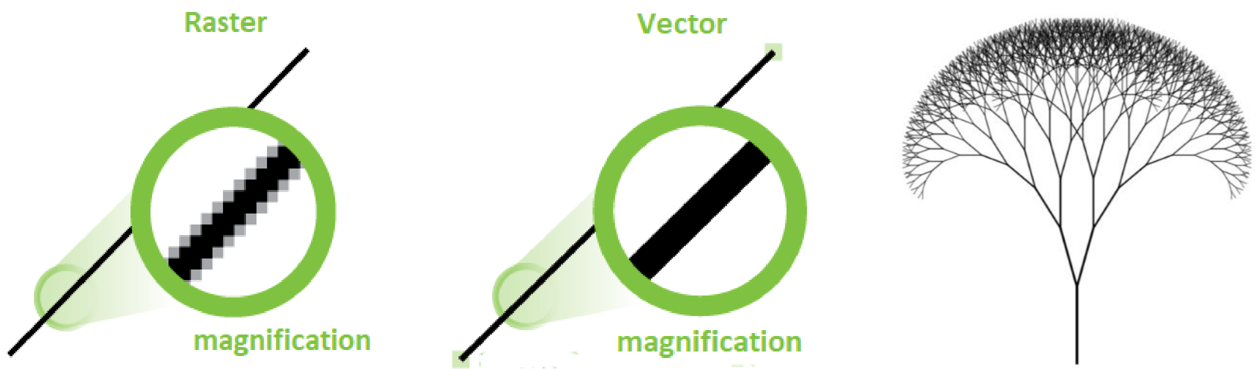
Fractals attract attention, fascinate, hypnotize. However, many believe that such images are simply patterns that are only good on the monitor screen or as applied aids for designing various printed products. In this case, few people realize that this simplicity is only apparent. In fact, fractal graphics is quite complex and the result of the fusion of mathematics and art. Today fractals are one of the most promising, rapidly developing types of computer graphics.

Before proceeding to the consideration of fractal graphics, we will consider the essence of computer, or "machine" graphics, as well as the generally accepted classification of computer graphics. This concept appeared relatively recently, in the 60s of the last century, when electronic computing devices were invented. The term "computer graphics" is interpreted in different sources in different ways. Some define it as an area of computer science, dealing with the issues of obtaining various images (drawings, drawings, animation) on a computer. Computer graphics encompasses all types and forms of presentation of images that are available for human perception on a monitor screen or as a copy on an external medium (paper, fabric, film, etc.). In other sources, computer graphics is called a special area of computer science that studies methods and tools for creating and processing images using software and hardware computing systems.

In a broad sense, computer graphics is all that uses a visual, imaginative display environment on a monitor. If you narrow the concept to practical use, then under computer graphics you can mean the process of creating, processing and displaying various kinds of images using a computer.

Depending on the method of imaging computer graphics is divided into raster, vector and fractal (Pic. 1).

The main and smallest element of the bitmap is a point. When an image is in a software environment on the screen, it is called a pixel. Each pixel of a bitmap image has two characteristics: placement and color. The greater the number of pixels and the smaller their size, the better the image looks. Large amounts of data are a major problem when using bitmaps. The second disadvantage of raster images relates to the impossibility of their increase for consideration of details. Since the image consists of dots, the magnification of the image leads to the fact that these dots become larger and resemble a mosaic, and therefore no further details can be considered in this case. Moreover, the increase in the points of the raster visually distorts the image and makes it grainy. This effect is called pixelation.



Pic. 1. Types of computer graphics: a - raster; b - vector; in - fractal

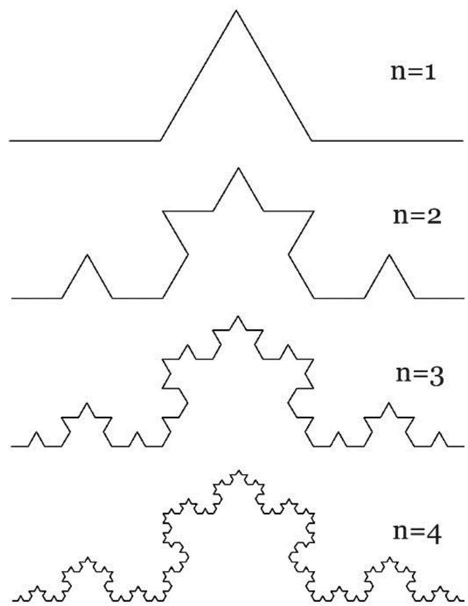
In vector graphics, the main element of the image is a line (whether it is a straight line or a curve). Of course, in the raster graphics there are also lines, but there they are considered as a combination of points. For each point of the line in the raster graphics, one or several memory cells are allocated (the more colors there are dots, the more cells they are allocated). Accordingly, the longer the raster line, the more memory it takes. In vector graphics, the amount of memory occupied by a line does not depend on the size of the line, since the line is represented as a formula, or rather, as several parameters. Whatever we do with this line, only its parameters that are stored in memory cells change. The number of cells for any line remains unchanged.

Pic. 2. An example of fractality in nature - Romanesco cabbage

The image in vector format is easily edited: it can be scaled, rotated, deformed without loss. Imitation of three-dimensionality in vector graphics is also easier than in raster. The fact is that each transformation is actually performed as follows: the old image (or a fragment) is erased, and a new one is built instead. The mathematical description of the vector picture remains the same - only the values of some variables, such as coefficients, change.

Fractal graphics are relatively young compared to raster and vector graphics. The basis of fractal graphics is fractal geometry, which allows to mathematically describe various types of inhomogeneities found in nature. The concepts of "fractal", "fractal geometry" and "fractal graphics" appeared in the late 1970s. The word "fractal" is derived from the Latin fractus and means "consisting of fragments." It was proposed by the mathematician Benoit Mandelbrot in 1975 to designate irregular, but self-similar structures. The birth of fractal geometry is usually associated with the release in 1977 of the book "The Fractal Geometry of Nature" by Benoit Mandelbrot. The definition of a fractal given by Mandelbrot: a fractal is a structure consisting of parts that are in some sense similar to the whole. Self-similarity is one of the basic properties of fractals. Thus, fractal graphics is a type of computer graphics in which self-similar structures (in other words, fractals) are used to some extent. Next, we will talk about what self-similarity is and where fractals occur in nature.

What is meant by self-similarity? The Romanesco cabbage from Italy is the most characteristic example of a fractal object in nature. Her cabbage buds grow in the form of a certain spiral (Pic. 2), which is called logarithmic, and the number of cabbage buds coincides with the Fibonacci number. The Fibonacci numbers are the elements of the numerical sequence 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946 ..., in which each successive number is equal to the sum of the two previous numbers. They received their name in honor of the medieval mathematician Leonardo of Pisa (known as Fibonacci). Each part of the elements of cabbage Romanesco has the same shape as the whole head. This property is repeated with regularity at different scales. In fact, this cabbage is a natural fractal. That is, no matter how we increase the fractal, after each step we will see the same shape that is characteristic of this fractal as a whole. Thus, two more concepts are closely related to fractals - iteration and recursion. Recursion is the process of repeating elements in a self-similar way. Iteration - simply speaking - the reapplication of a mathematical operation.



Pic. 3. Koch Curve Recursion

In fact, fractal properties have a very large number of natural objects - just few people think about it. You can admire the clouds in the sky, incidental waves of the surf, walk in the woods - and not even suspect that mathematics is at the heart of this beauty! Benoit Mandelbrot began to conduct studies of the fractal properties of natural objects. It turns out that despite the complexity of natural objects, many of them are, in principle, described by fairly simple mathematical formulas. Although pure fractals do not exist in nature. What we are seeing is the so-called stochastic fractals. That is, such fractals, which are obtained if in the iterative process randomly change any of its parameters. The “pure” fractal can be brought to infinity, since it has infinite recursion, but this cannot be said about stochastic fractals.

It should be noted that the word "fractal" is not a mathematical term and does not have a generally accepted rigorous mathematical definition. It can be used when the figure in question has any of the following properties:

• has a non-trivial structure on all scales - this is how a fractal differs from regular shapes (such as a circle, an ellipse, a smooth function graph): if we look at a small fragment of a regular figure on a very large scale, then it will look like a straight line fragment. For a fractal, an increase in scale does not lead to a simplification of the structure, therefore on all scales we will see an equally complex picture;

• is self-similar or approximately self-similar;

• has a fractional metric dimension or metric dimension that exceeds the topological one.

In addition, to build a fractal, it is necessary to take into account the initial state and the formula describing it. This is called initial set, which is passed through a mechanism that causes its display and adds the displayed set to the original one. This process is called iteration. Thus, after several similar relatively simple operations, a very complex image is obtained. In the process of obtaining a fractal, two things are important: the initial set and the transformation mechanism. Depending on the algorithm for constructing fractals are divided into linear and nonlinear.

Algorithms for constructing linear fractals are determined by linear functions. Self-similarity in them is present in the simplest version: every part repeats the whole.

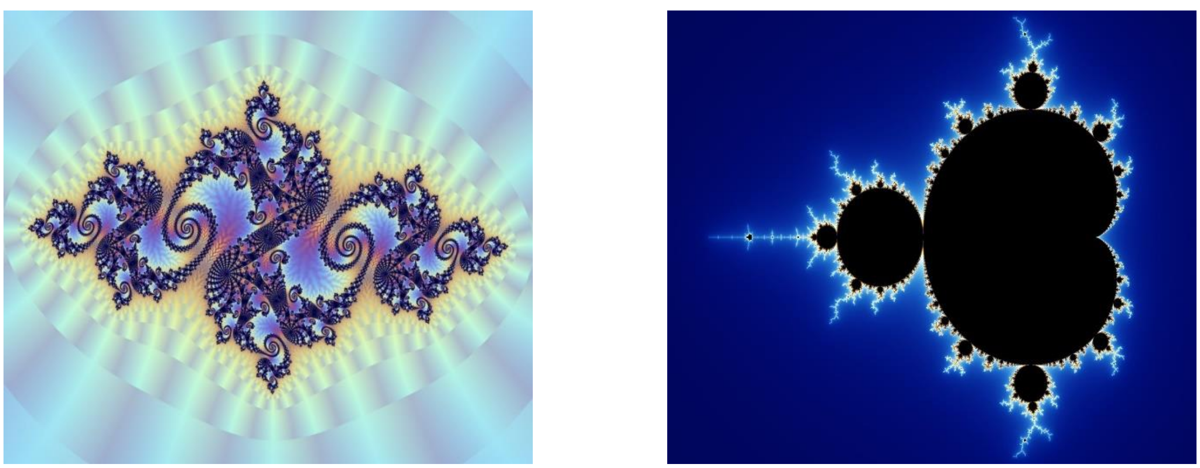
Nonlinear fractals are given by a nonlinear growth function, that is, equations to a degree above the first. Self-similarity in them will be more complicated: any part is no longer an exact, but a deformed copy of the whole.

One of the simplest examples of a linear fractal is the Koch curve (1904, the German mathematician Helga von Koch).

There is a simple recursive procedure (obtaining self-similar parts of a fractal) of forming fractal curves on a plane. We define an arbitrary polyline with a finite number of links, which are called the generator. Next, we replace each segment with a generator (more precisely, a broken line, similar to a generator). In the resulting broken line, we again replace each segment with a generator. Continuing to infinity, in the limit, we obtain a fractal curve. Pic. 3 shows several steps of this procedure for the Koch curve.

One of the first non-linear fractals was described by the French mathematician Gaston Julia back in 1918. But in his work, there were no images of the sets he studied and the term “fractal”.

Nowadays, computers allowed us to obtain images of Julia sets (Pic. 4a), which, together with the Mandelbrot Sets (Pic. 4b), are now the most famous quadratic fractal structures.



Pic. 4. Images of Julia (a) and Mandelbrot sets (b)

Both types of fractals result from the implementation of the simplest nonlinear algorithm on the complex plane.

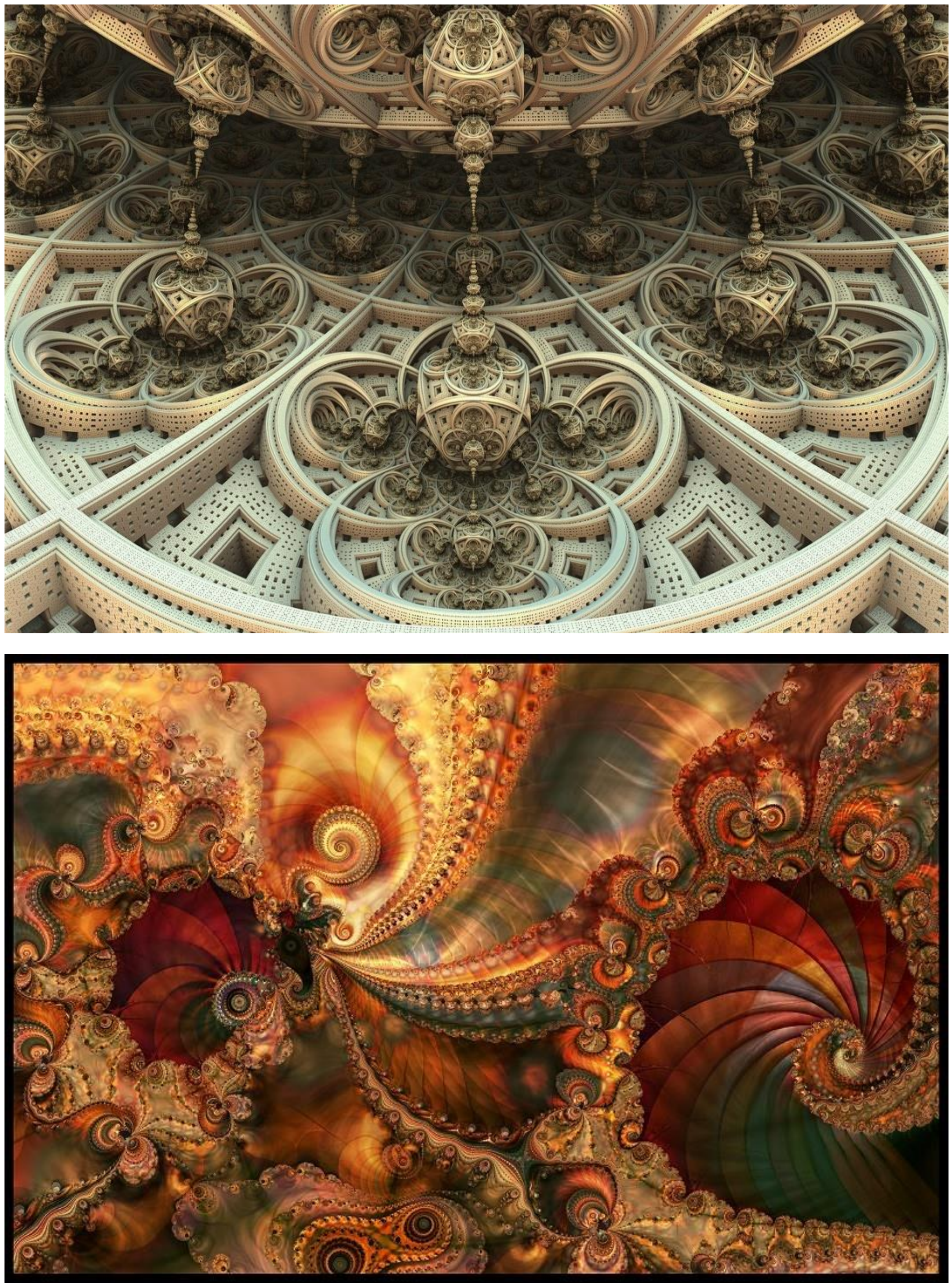
Here, the method of imaging is based on the principle of inheritance from the so-called parents of the geometric properties of the heir objects. The construction of a fractal pattern is carried out according to some algorithm or by automatically generating images using calculations using specific formulas. Changes in the values in the algorithms or the coefficients in the formulas leads to the modification of these images. The main advantage of fractal graphics is that only algorithms and formulas are saved in the fractal image file.

A fractal is an object whose individual elements inherit the properties of the parent structures. Since a more detailed description of the elements of a smaller scale occurs according to a simple algorithm, such an object can be described with just a few mathematical equations.

Fractals allow you to describe entire classes of images, for a detailed description of which it requires relatively little memory. At the same time, fractals are poorly applicable to images outside of these classes.

Software tools for working with fractal graphics are designed to automatically generate images by mathematical calculations. That is why the fractal graphics are not recognized by either computer or ordinary artists due to the fact that the program supposedly does everything for a person. In fact, the process of working with fractal graphics, although automated, is nevertheless completely creative: combining formulas and changing variables, one can achieve amazing results and embody the most daring artistic designs. Creating a fractal artistic composition is not in drawing or design, but in programming.

By changing and combining the colors of fractal figures, it is possible to model images of animate and inanimate nature (for example, branches of a tree or snowflakes), as well as to compose a “fractal” composition from the obtained figures. Fractal graphics, as well as vector and 3D, are computed. Its main difference is that the image is constructed by an equation or a system of equations. Therefore, to perform all the calculations in the computer's memory nothing is stored except the formula.



Pic. 5. Images obtained by using fractal generators

Only by changing the coefficients of the equation, you can get a completely different image. This idea has been used in computer graphics due to the compactness of the mathematical apparatus necessary for its implementation. So, with the help of several mathematical coefficients, you can define lines and surfaces of a very complex shape.

In computer graphics, fractal geometry is indispensable for generating artificial clouds, mountains, and the surface of the sea. In fact, thanks to fractal graphics, a method has been found for efficiently implementing complex non-Euclidean objects, the images of which are very similar to natural ones. Therefore, this article is given such a name. Many natural objects have fractal properties, so they are easy to be created on a computer by using fractal graphics. For example, when developing a computer game, there is no need to re-draw a forest, mountains, clouds, etc. These objects are self-similar and, therefore, they can be easily generated with software by using mathematical formulas. Adding or changing some parameters of the original formula, you can achieve an amazing diversity of the obtained natural objects. Fractals on a computer screen are patterns built by the PC itself according to a given program. In addition to fractal painting, there are fractal animation and music.

In conclusion, I would like to note the following: fractal graphics is one of the most unusual and promising areas in computer graphics. The results that can be obtained with its help amaze the imagination of even the most sophisticated connoisseurs of computer art. Thus, the images created with the help of software fractal generators sometimes contain fantastic and unusual landscapes (Pic. 5), which the surrealists did not even dream of. And vice versa, with the help of fractal graphics it is possible to depict with amazing accuracy what we see in the world around us. Truly the world of fractals is amazing!

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